Swami Ramanand Teerth Marathwada University

Department of Mathematics

Research Proposal

for

Doctor of Philosophy (Ph.D.)in Mathematics

Title:

Fractional Order Differential and Integral Inclusions in Banach Algebras

Submitted by

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1. Introduction

Fractional calculus is the branch of mathematics, which deals with the study of integral and differential equations and inclusions of arbitrary order, are an important and efficient tool for the study of dynamical systems. Fractional order models are found to be more adequate. The idea of fractional calculus has been known since the development of the classical calculus, with the first reference probably being associated with Leibniz and L'Hospitals where they discussed about half order derivative.

In some real problems as fractional order inclusionsions provides a good discussion of properties of varies materials and processes. On the other hand number of problems in mathematics such as control theory, Mathematical economics, game theory etc. requires models that are described by inclusions. This leads to mathematical models that contain inclusions. consider the work has been done over the integers order inclusions by different authors N.S. Papageorgiou [15], Bashir Ahmad [3], R.P. Agarwal[17]etc. Fractional order model found to to be more adequet then integer model. There is an in depth development of the theory of fractional order inclusions that gives rise to a great work.

The studies on fractional order inclusions is restricted to the existence theory only for particular classes of nonlinear inclusions and fixed point theorems; however other aspects such as quadratic, functional inclusions of fractional order for such systems, existence of extremal solutions, attractivity results of the solutions are of great interest. In the subject of fractional order differential equation gains much more importance and attention. for details and examples see[3] some results are available in the literature on existence of solutions.

2. Reviews of Literature and development in the subject

The study of fractional order inclusions involves mathematical modeling describing the dynamical system. The integral and differential inclusion in finite dimensional context has been properly studied in the literature.The initial stimulation for the study of inclusions can be seen in the papers and thesis. some recent work on this topics can be seen in the papers of N.S Papageorgiou [15], B.C.Dhage[4, 5],D.S palimkar, A.F.Siddiqui [6, 7, 8]. In many dynamical system can be modelled by fractional inclusions are troublously investigated by many authors see the papers of R.P.Agarwal[17], B.Ahmad and S.K Ntouyas[3], M.Benchohra[12], G.V Smirnov[9] and reference therein. the study of fractional order inclusions is rare in literature.

3. Objectives of research /proposed Hypothesis:-

The study will be creating new idea and contribution to the theory of fractional order inclusions. The aims of the present study of the following points:-

1) To studysome known fractional order differential, integral and integro-differential inclusions with initial conditions on bounded and unbounded intervals.

2) To study some fractional order differential, integral and integro-differential inclusions with boundary conditions on bounded as well as unbounded intervals.

3) To study existence the solutions as well as existence the extremal solutions for some known and unknown integer order and fractional order functional, quadratics etc. inclusions for current concept by fixed point theory.

4)To study the existence of attractive results for fractional order inclusions.

5)To discuss the inclusions of first, second and fractional order for their different aspects of solutions of bounded and unbounded

intervals of real line.
6) To discuss some applications of the integral and differential inclusions of fractional order studied in the present work of some concrete problems of the other areas of mathematics.
7)The main tools employed in our analysis is based on the theory of fractional calculus and fixed point theorems.
In this study we will also discuss and improve some known and unknown techniques of existence of solution for fractional order inclusion. we will also solve some problem and calculating solutions for fractional order inclusions.

4. Methodology to be adopted

There are different methods for the existence of solution of known and unknown fractional order inclusions. following are some methods that can be adopted.

(1) Adomain decomposition method
(2) Approximation Method
(3) Measurable selection method
(4) Numerical Method
(5) Iteration Method
(6) The Laplace transform Method
(7) The Series Solution Method
(8) Ranga-Kutta Method
(9) The fixed point method etc.

From the above methods we will be using one of them. The fixed point method is a powerful device for existence of the solution. The application has increased through the development of accurate and enough techniques for computing fixed point method literature.

The monotone technique for the different types of fractional order inclusions is also very useful. The use of monotone technique

in the theory of fractional order inclusions involving the discontinuous multi-valued functions is relatively new to the literature.

5. Importance of study/Application to society

The theory of inclusions occupies an important role in the development of Mathematics.The subject is gaining much attention due to it's vide application in the field of physics, chemistry, Electrodynamics, complex medium. It involves different types of fractional order inclusions. The subject of fractional order inclusions is useful in control system, mathematical economics, game theory and bio-engineering etc. Inclusions play an important role as a tool in the study of chaotic system. There are two different pairs of fractional order systems. choatic system as the master system. A very important role not in mathematics but also in physics, dynamical system and engineering.

In the study of fractional calculus, they are also very useful in providing existence of fractional order inclusion. The research interests of the author are in fluid mechanics, nonlinear waves and mathematical models of biology.

6. proposed work plan /formulation and structure of study:-

First year plan:-

The firstly, I will be start with surveying the work done in this area and study the collecting material available in the lerature and upon that wI'll build up a base to know and understand the area of research. Some research papers, journals and books related to research area will be studied. In this year I will attend some seminars/ conferences and also complete the course work. To visit different institutes libraries using internet to collect the references on the proposed research topic.

Second year plan:-

By collected material, I will discover the direction of research, methodology, tools and techniques. New research problems are to be formulated and an attempt to solve it. Now taking Consider fractional order hybrid quadratic functional differential inclusion.

$$D^{\alpha}[\frac{x(t)-x(0)}{g(t,x(\phi_2(t)))}] \in f(t,x(\phi_1(t))), t \in \mathbb{R}_+ \tag{1}$$

Multivalued nonlocal such that the integral condition for first and second order $x(0) = 0$ and fractional order integral condition with $x_0 = 0$

$$x(0) - x_0 \in \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} H(s,x(\phi_3(s)))ds\beta \leq \alpha, t \in \mathbb{R}_+ \tag{2}$$

D^{α} is Riemann derivative of $\alpha \in (0,1)$,$f : \mathbb{R}_+ \times \mathbb{R} \to P(\mathbb{R})$, $g : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R} - 0$, $H : \mathbb{R}_+ \times \mathbb{R} \to P(\mathbb{R})$, $\phi_1, \phi_2, \phi_3 : \mathbb{R}_+ \to \mathbb{R}_+$ $\mathbb{R}_+$ are two set valued mapping where $P(\mathbb{R})$ denote family of nonempty subset of $\mathbb{R}$.

Solving such problems with existence of extremal solution and give some examples based on it. In this stage I will be submitted to research journal for publication. Different techniques will be solved and it will submitting to the research journal for publication this year improvement of research work and verification of the methodology adopted.

Third year plan :-

In the third year, we solve another differential inclusion. Consider fractional order hybrid quadratic differential inclusion.

$$D^{\alpha}(\frac{x(t)-\sum_{i=1}^{n} q_i(t,x(t))}{g(t,x(t))}) \in f(t,x(t))$$

$$x(0) = 0, t \in \mathbb{R}^+$$

Where D^{α} is Riemann derivative of order α and $f : \mathbb{R}^{+} \times \mathbb{R} \to P(\mathbb{R})$ is multivalued function $g : \mathbb{R}^{+} \times \mathbb{R} \to \mathbb{R} - 0$ and $q_i : \mathbb{R}^{+} \times \mathbb{R} \to \mathbb{R}$ are two set valued mapping and $q_i(0,0) = 0$ for $i = 1, 2, 3, ...n$ where $P(\mathbb{R})$ denote family of non-empty subset of $\mathbb{R}$ solving such problems with existence of extremal solution with examples. I will be presented in national and international gathering such as seminars,workshops, conferences The research work will be published in the several journals and thesis writing up the finding in the research work.

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